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MODIFICATIONS OF THE TEST INFORMATION FUNCTION

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A minimum bound of any estimator, biased or unbiased, is considered, and, based on that, Modification Formula No. 1 is proposed for the maximum likelihood estimator, in place of the test information function. A minimum bound of the mean squared error is considered, and, based on that, Modification Formula No. 2 in the same context is proposed. Examples are given, and the usefulnesses of these modified test information functions in computerised adaptive testing are discussed. These topics are also discussed and observed for the monotonically transformed latent variable.

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The research was conducted at the principal investigator's laboratory, 405 Austin Peay Bldg., Department of Psychology, University of Tennessee, Knoxville, Tennessee. Those who worked as assistants for this research include Christine A. Golik, Barbara A. Livingston, Lee Hai Gan and Nancy H. Domm.

I Introduction

In latent trait models and their applications, the test information function has an important role, and has proved to be useful in many ways. Let θ be ability, or latent trait, which assumes any real number. We assume that there is a set of n test items measuring θ whose characteristics are known. Let g denote such an item, k_g be a discrete item response to item g, and $P_{k_g}(\theta)$ denote the operating characteristic of k_g, or the conditional probability assigned to k_g, given θ, i.e.,

$$(1.1) \qquad P_{k_g}(\theta) = prob.[k_g \mid \theta] \ .$$

We assume that $P_{k_g}(\theta)$ is three-times differentiable with respect to θ. We have for the *item response information function* (Samejima, 1972)

$$(1.2) \qquad I_{k_g}(\theta) = -\frac{\partial^2}{\partial\theta^2}\log P_{k_g}(\theta) = [\frac{\partial}{\partial\theta}P_{k_g}(\theta)\ \{P_{k_g}(\theta)\}^{-1}]^2 - \frac{\partial^2}{\partial\theta^2}P_{k_g}(\theta)\ [P_{k_g}(\theta)]^{-1} \ ,$$

and the *item information function* is defined as the conditional expectation of $I_{k_g}(\theta)$, given θ, such that

$$(1.3) \qquad I_g(\theta) = E[I_{k_g}(\theta) \mid \theta] = \sum_{k_g} I_{k_g}(\theta)P_{k_g}(\theta) = \sum_{k_g} [\frac{\partial}{\partial\theta}P_{k_g}(\theta)]^2[P_{k_g}(\theta)]^{-1} \ .$$

In the special case where the item g is scored dichotomously, this item information function is simplified to become

$$(1.4) \qquad I_g(\theta) = [\frac{\partial}{\partial\theta}\ P_g(\theta)]^2\ [\{P_g(\theta)\}\{1 - P_g(\theta)\}]^{-1} \ ,$$

where $P_g(\theta)$ denotes the operating characteristic of the correct answer to item g.

Let V be a response pattern such that

$$(1.5) \qquad V = \{\ k_g\ \}' \qquad g = 1, 2, ..., n \ .$$

The operating characteristic, $P_V(\theta)$, of the response patten V is defined as the conditional probability of V, given θ, and by virtue of *local independence* we can write

$$(1.6) \qquad P_V(\theta) = \prod_{k_g \epsilon V} P_{k_g}(\theta) \ .$$

The *response pattern information function,* $I_V(\theta)$, (Samejima, 1972) is given by

$$(1.7) \qquad I_V(\theta) = -\frac{\partial^2}{\partial\theta^2}\log P_V(\theta) = \sum_{k_g \epsilon V} I_{k_g}(\theta) \ ,$$

and the *test information function,* $I(\theta)$, is defined as the conditional expectation of $I_V(\theta)$, given θ, and we obtain from (1.2), (1.3), (1.5), (1.6) and (1.7)

$$(1.8) \qquad I(\theta) = E[I_V(\theta) \mid \theta] = \sum_V\ I_V(\theta)P_V(\theta)\ = \sum_{g=1}^{n} I_g(\theta) \ .$$

It is a big advantage of the modern mental test theory over classical mental test theory that the standard error of estimation can *locally* be defined by means of $[I(\theta)]^{-1/2}$, which *does not depend upon the population of examinees,* but is solely a property of the test itself. In computerized adaptive testing, for example, this function can be used for the stopping rule indicating the desirable accuracy of estimation of the examinee's ability (cf. Samejima, 1977b), provided that our itempool contains a large number of items whose difficulty levels distribute widely over the range of θ of interest.

In a case where our test does not have a large amount of information for the entire range of θ of interest, however, we may have some reservations in using $[I(\theta)]^{-1/2}$ as a measure of local accuracy of estimation for all θ .

It has been shown (Samejima, 1977a, 1977b) that in many cases the conditional distribution of $\hat{\theta}_V$, given θ , converges to $N(\theta,[I(\theta)]^{-1/2})$ relatively quickly. On the other hand, we have also noticed that the speed of convergence is not the same even if the amount of test information is kept equal. This has been demonstrated by using Constant Information Model (Samejima, 1979a), which is represented by

$$P_g(\theta) = \sin^2[a_g(\theta - b_g) + (\pi/4)] \quad , \tag{1.9}$$

where, as before, $P_g(\theta)$ denotes the operating characteristic of the correct answer, and a_g (> 0) and b_g are the item discrimination and difficulty parameters, respectively. This model provides us with a constant amount of item information $I_g(\theta)$ which equals $4a_g^2$ for the interval of θ ,

$$-\pi[4a_g]^{-1} + b_g < \theta < \pi[4a_g]^{-1} + b_g \quad . \tag{1.10}$$

For the purpose of illustration, Figures 1-1 and 1-2 present part of the results obtained by using Monte Carlo studies (cf. Samejima, 1979b). In this study, twenty hypothetical tests of ten to two hundred equivalent items with the common parameters, $a_g = 0.25$ and $b_g = 0.00$, were administered to one hundred examinees hypothesized at each of the eight different levels of θ , i.e., $\theta = -3.0, -2.2, -1.4, -0.6, 0.2, 1.0, 1.8, 2.6$. Thus these items provide us with the same amount of constant item information, 0.25 , for the interval of θ ,

$$-\pi < \theta < \pi \quad . \tag{1.11}$$

In these figures, the results of ten and twenty items for $\theta = 0.2$ and for $\theta = 2.6$ are shown, respectively. In each graph, the cumulative frequency ratio of the maximum likelihood estimates $\hat{\theta}_V$ *'s* of the one hundred hypothetical examinees, the asymptotic normal distribution function $N(\theta,[I(\theta)]^{-1/2})$ and the normal distribution function using the sample mean and standard deviation of the one hundred $\hat{\theta}_V$ *'s* as the two parameters are presented. These figures indicate that, even when the number of items is as small as 20 or 10 , the normal approximation of the distribution of the maximum likelihood estimate $\hat{\theta}_V$ works well when the level of θ is close to the common difficulty parameter b_g , while the convergence is much slower when the level of θ is far away from b_g . Note that in both examples the amounts of test information are the same, i.e., $I(\theta;\theta = 0.2) = I(\theta;\theta = 2.6)$, which equals 2.5 for the ten item case and 5.0 for the twenty item case.

The above examples indicate that in certain situations *a test does not provide us with as much amount of test information as its test information function makes us believe* on certain levels of ability θ . This fact suggests that we need to be careful, and in some cases we may need some modification of the test information function.

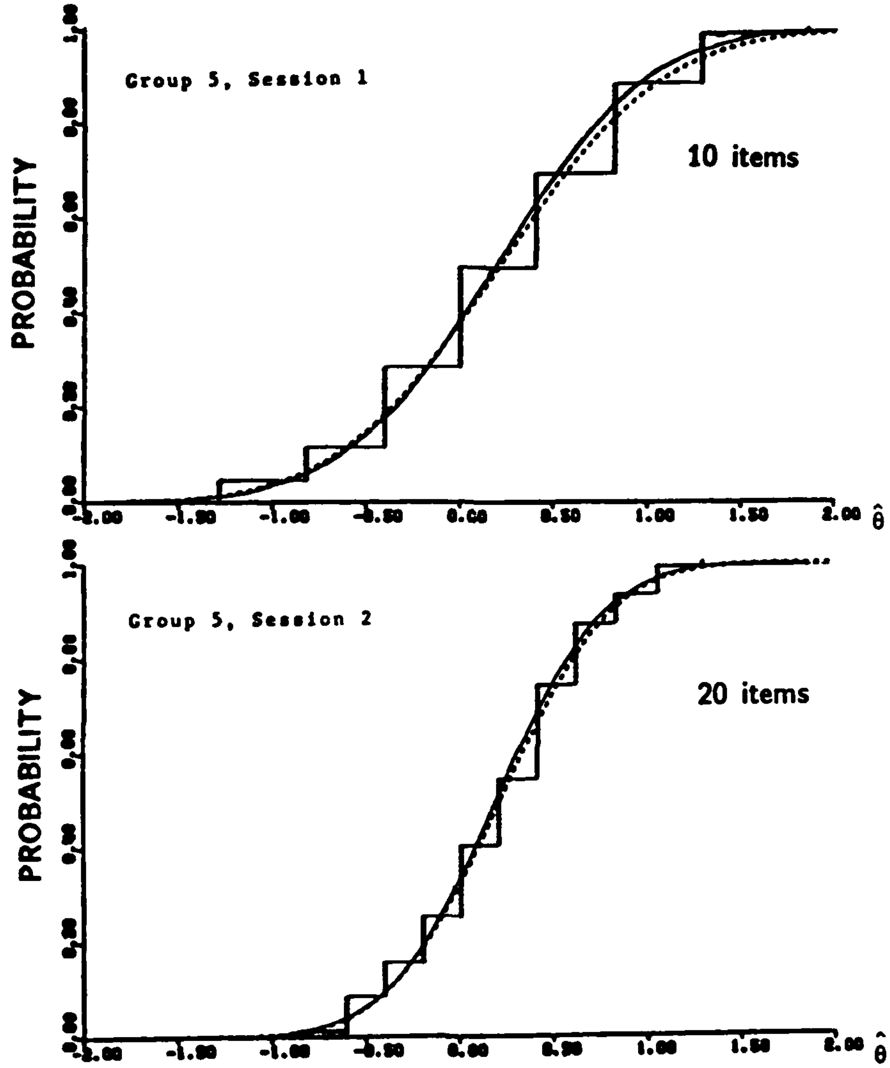


FIGURE 1-1

Cumulative Frequency Ratio of $\hat{\theta}_V$'s of the One Hundred Hypothetical Examinees with Ability Level, $\theta = 0.2$, (Step Function), the Asymptotic Normal Distribution Function $N(\theta, [I(\theta)]^{-1/2})$ (Solid Line) and the Normal Distribution Function Using the Sample Mean and Standard Deviation of $\hat{\theta}_V$'s As the Two Parameters (Dotted Line). Ten and Twenty Equivalent Items Following the Constant Information Model with $a_g = 0.25$ and $b_g = 0.00$ Were Administered, Respectively, in the Two Separate Sessions.

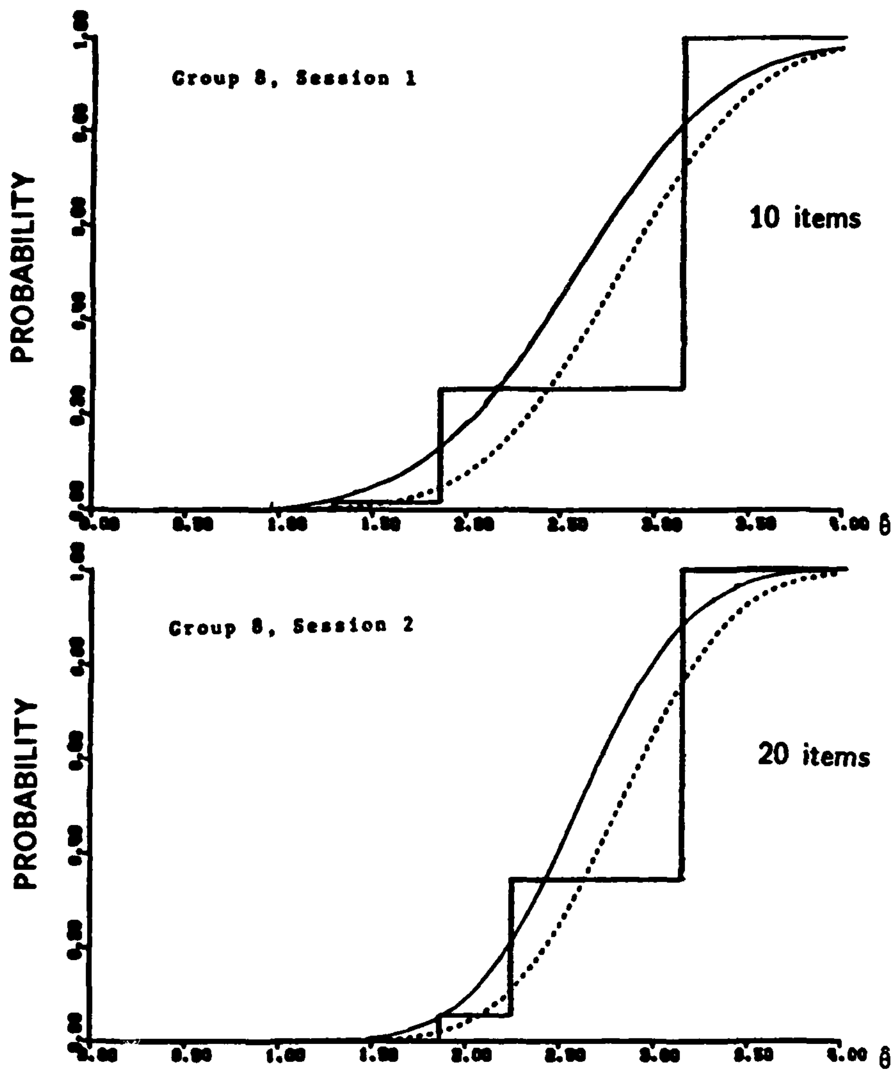


FIGURE 1-2

Cumulative Frequency Ratio of $\hat{\theta}_V$'s of the One Hundred Hypothetical Examinees with Ability Level, $\theta = 2.6$, (Step Function), the Asymptotic Normal Distribution Function $N(\theta,\ [I(\theta)]^{-1/2})$ (Solid Line) and the Normal Distribution Function Using the Sample Mean and Standard Deviation of $\hat{\theta}_V$'s As the Two Parameters (Dotted Line). Ten and Twenty Equivalent Items Following the Constant Information Model with $a_g = 0.25$ and $b_g = 0.00$ Were Administered, Respectively, in the Two Separate Sessions.

The present paper proposes two modification formulae of the test information function $I(\theta)$, in order to provide better measures of local accuracies of the estimation of θ, when the maximum likelihood estimation is used to provide us with the estimate of ability θ.

II Minimum Variance Bound

The reciprocal of the test information function $I(\theta)$ also provides us with a minimum variance bound for any *unbiased* estimator of θ (cf. Kendall and Stuart, 1961). Since the maximum likelihood estimate, which is denoted by $\hat{\theta}_V$, is only *asymptotically unbiased*, for a finite number of items we need to examine if the bias of $\hat{\theta}_V$ of a given test over the meaningful range of θ is practically nil, before we consider this reciprocal as a minimum variance bound. In this section we shall consider a minimum variance bound which applies for any estimator of θ, *biased or unbiased.*

Let θ_V^* denote any estimator of θ. We can write in general

(2.1) $$E(\theta_V^* \mid \theta) = \theta + E[(\theta_V^* - \theta) \mid \theta] \ .$$

When the item responses are discrete, we have

(2.2) $$E(\theta_V^* \mid \theta) = \sum_V \theta_V^* \ L_V(\theta) = \sum_V \theta_V^* \ P_V(\theta) \ \ ,$$

where $L_V(\theta)$ denotes the likelihood function. Differentiating both sides of (2.2) with respect to θ, we obtain

(2.3) $$\begin{aligned} \frac{\partial}{\partial\theta} E(\theta_V^* \mid \theta) &= \frac{\partial}{\partial\theta} [\sum_V \theta_V^* \ P_V(\theta)] = \sum_V \theta_V^* [\frac{\partial}{\partial\theta} P_V(\theta)] \\ &= \sum_V [\theta_V^* - E(\theta_V^* \mid \theta)] \, [\frac{\partial}{\partial\theta} P_V(\theta)] \, , \end{aligned}$$

since we have

(2.4) $$\sum_V P_V(\theta) = 1 \ ,$$

(2.5) $$\sum_V [\frac{\partial}{\partial\theta} P_V(\theta)] = 0$$

and

(2.6) $$\sum_V E(\theta_V^* \mid \theta) \, [\frac{\partial}{\partial\theta} P_V(\theta)] = E(\theta_V^* \mid \theta) \, \sum_V [\frac{\partial}{\partial\theta} P_V(\theta)] = 0 \ .$$

We can write

(2.7) $$\frac{\partial}{\partial\theta} P_V(\theta) = [\frac{\partial}{\partial\theta} \log P_V(\theta)] \ P_V(\theta) \ \ ,$$

and using this we can rewrite (2.3) into the form

$$\frac{\partial}{\partial\theta}E(\theta_V^* \mid \theta) = \sum_V [\theta_V^* - E(\theta_V^* \mid \theta)] \, [\frac{\partial}{\partial\theta}\log P_V(\theta)] \, P_V(\theta) \quad . \tag{2.8}$$

From this result, by the Cramér-Rao inequality, we obtain

$$[\frac{\partial}{\partial\theta}E(\theta_V^* \mid \theta)]^2 \le Var.(\theta_V^* \mid \theta) \; E[\{\frac{\partial}{\partial\theta}\log P_V(\theta)\}^2 \mid \theta] \quad . \tag{2.9}$$

Since we can write

$$E[\{\frac{\partial}{\partial\theta}\log L_V(\theta)\}^2 \mid \theta] = -E[\frac{\partial^2}{\partial\theta^2}\log L_V(\theta) \mid \theta] \quad , \tag{2.10}$$

from this, (1.7), (1.8) and (2.1) we can rewrite and rearrange the inequality (2.9) into the form

$$Var.(\theta_V^* \mid \theta) \ge [\frac{\partial}{\partial\theta}E(\theta_V^* \mid \theta)]^2 \, [I(\theta)]^{-1} = [1 + \frac{\partial}{\partial\theta}E(\theta_V^* - \theta \mid \theta)]^2 \, [I(\theta)]^{-1} \quad . \tag{2.11}$$

The rightest hand side of (2.11) provides us with the minimum variance bound of the conditional distribution of any estimator θ_V^* . When θ_V^* is an unbiased estimator of θ , the second term of the first factor equals zero, and we obtain the reciprocal of the test information function for the minimum variance bound. When θ_V^* is biased, however, the size of the minimum variance bound is determined by this second term, and it can be greater or less than the reciprocal of the test information function depending upon the sign of this partial derivative.

III First Modified Test Information Function

Lord has proposed a bias function for the maximum likelihood estimate of θ in the three-parameter logistic model whose operating characteristic of the correct answer, $P_g(\theta)$, is given by

$$P_g(\theta) \;=\; c_g + (1 - c_g)[1 + \exp\{-Da_g(\theta - b_g)\}]^{-1} \quad , \tag{3.1}$$

where a_c , b_g , and c_g are the item discrimination, difficulty, and guessing parameters, and D is a scaling factor, which is set equal to 1.7 when the logistic model is used as a substitute for the normal ogive model. Lord's bias function $B(\hat\theta_V \mid \theta)$ can be written as

$$B(\hat\theta_V \mid \theta) \;=\; D[I(\theta)]^{-2}\sum_{g=1}^{n} a_g I_g(\theta)[\psi_g(\theta) - \frac{1}{2}] \quad , \tag{3.2}$$

where

$$\psi_g(\theta) \;=\; [1 + \exp\{-Da_g(\theta - b_g)\}]^{-1} \tag{3.3}$$

(cf. Lord, 1983). We can see in the above formula of the MLE bias function that the bias should be negative when $\psi_g(\theta)$ is less than 0.5 for all the items, which is necessarily the case for lower values of θ , and should be positive when $\psi_g(\theta)$ is greater than 0.5 for all the items, i.e., for higher values of θ , and in between the bias tends to be close to zero, for the last factor in the formula assumes negative values for some items and positive values for some others, provided that the difficulty parameter b_g

distributes widely. Lord has applied this MLE bias function for an 85-item SAT Verbal test (Lord, 1984), and the result shows a wide range of θ in which the bias is practically nil.

In the general case of discrete item responses, we obtain for the bias function of the maximum likelihood estimate (cf. Samejima, 1987)

$$(3.4) \qquad B(\hat{\theta}_V \mid \theta) = E[\hat{\theta}_V - \theta \mid \theta] = -(1/2)[I(\theta)]^{-2}\sum_{g=1}^{n}\sum_{k_g} A_{k_g}(\theta)P''_{k_g}(\theta)$$

$$= -(1/2)[I(\theta)]^{-2}\sum_{g=1}^{n}\sum_{k_g} P'_{k_g}(\theta)P''_{k_g}(\theta)[P_{k_g}(\theta)]^{-1} \ ,$$

where $A_{k_g}(\theta)$ is the basic function for the discrete item response k_g , and $P'_{k_g}(\theta)$ and $P''_{k_g}(\theta)$ denote the first and second partical derivatives of $P_{k_g}(\theta)$ with respect to θ , respectively. On the graded response level where item score x_g assumes successive integers, 0 through m_g , each k_g in the above formula must be replaced by the graded item score x_g (cf. Samejima, 1969, 1972). On the dichotomous response level, it can be reduced to the form

$$(3.5) \qquad B(\hat{\theta}_V \mid \theta) = E[\hat{\theta}_V - \theta \mid \theta] = (-1/2)[I(\theta)]^{-2}\sum_{g=1}^{n} I_g(\theta)P''_g(\theta)[P'_g(\theta)]^{-1} \ ,$$

with $P'_g(\theta)$ and $P''_g(\theta)$ indicating the first and second partial derivatives of $P_g(\theta)$ with respect to θ , respectively. This formula includes Lord's bias function in the three-parameter logistic model as a special case.

We can rewrite the inequality (2.11) for the maximum likelihood estimate $\hat{\theta}_V$

$$(3.6) \qquad Var.(\hat{\theta}_V \mid \theta) \geq [1 + \frac{\partial}{\partial\theta}B(\hat{\theta}_V \mid \theta)]^2 \ [I(\theta)]^{-1} \ .$$

Taking the reciprocal of the right hand side of (3.6), which is an approximate minimum variance bound of the maximum likelihood estimator, a modified test information function, $\Upsilon(\theta)$, can be defined by

$$(3.7) \qquad \Upsilon(\theta) = I(\theta) \ [1 + \frac{\partial}{\partial\theta}B(\hat{\theta}_V \mid \theta)]^{-2} \ .$$

From this formula, we can see that the relationship between this new function and the original test information function depends upon the first derivative of the MLE bias function. To be more precise, if the derivative is positive, then the new function will assume a lesser value than the original test information function. If it is negative, then this relationship will be reversed. If it is zero, i.e., if the MLE is unbiased, then these two functions will assume the same value. We can write from (3.4) for the general form of the derivative of the MLE bias function

$$(3.8) \quad \frac{\partial}{\partial\theta}B(\hat{\theta}_V \mid \theta) = \{I(\theta)\}^{-1}[(1/2)\{I(\theta)\}^{-1}\sum_{g=1}^{n}\sum_{k_g}(I_{k_g}(\theta)P''_{k_g}(\theta) - P'_{k_g}(\theta)P'''_{k_g}(\theta)\{P_{k_g}(\theta)\}^{-1})$$

$$-2B(\hat{\theta}_V \mid \theta)I'(\theta)] \ ,$$

where $P'''_{k_g}(\theta)$ and $I'(\theta)$ denote the third and the first derivatives of $P_{k_g}(\theta)$ and $I(\theta)$ with respect to θ , respectively. It is obvious from (1.3) and (1.8) that we have

(3.9)
$$I_g'(\theta) = \sum_{k_g} P_{k_g}'(\theta)[P_{k_g}''(\theta)\{P_{k_g}(\theta)\}^{-1} - I_{k_g}(\theta)]$$

and

(3.10)
$$I'(\theta) = \sum_{g=1}^{n} I_g'(\theta) = \sum_{g=1}^{n}\sum_{k_g} P_{k_g}'(\theta)[P_{k_g}''(\theta)\{P_{k_g}(\theta)\}^{-1} - I_{k_g}(\theta)] ,$$

where $I_g'(\theta)$ is the first derivative of the item information function $I_g(\theta)$ with respect to θ . For a set of dichotomous items (3.8) becomes simplified into the form

(3.11)
$$\begin{aligned}\frac{\partial}{\partial\theta}B(\hat{\theta}_V \mid \theta) &= \{I(\theta)\}^{-1}[(1/2)\{I(\theta)\}^{-1}\sum_{g=1}^{n}\{P_g(\theta)\}^{-2}\{1-P_g(\theta)\}^{-2} \\ &\quad (\{1-2P_g(\theta)\}\{P_g'(\theta)\}^2 P_g''(\theta) - P_g(\theta)\{1-P_g(\theta)\}(\{P_g''(\theta)\}^2 + P_g'(\theta)P_g'''(\theta))) \\ &\quad -2B(\hat{\theta}_V \mid \theta)\, I'(\theta)] ,\end{aligned}$$

where $B(\hat{\theta}_V \mid \theta)$ is given by (3.5).

IV Minimum Bound of the Mean Squared Error

When the estimator θ_V^* is conditionally *biased*, however small the conditional variance may be, it does not reflect the accuracy of estimation of θ . Thus the *mean squared error*, $E[(\theta_V^* - \theta)^2 \mid \theta]$, becomes a more important indicator of the accuracy. We can write for the mean squared error

(4.1)
$$E[(\theta_V^* - \theta)^2 \mid \theta] = Var.(\theta_V^* \mid \theta) + [E(\theta_V^* \mid \theta) - \theta]^2$$

(cf. Kendall and Stuart, 1961). We can see in this formula that the mean squared error equals the conditional variance if θ_V^* is unbiased, and is greater than the variance when θ_V^* is biased. From this and the inequality (2.11) we obtain for the minimum bound of the mean squared error

(4.2)
$$E[(\theta_V^* - \theta)^2 \mid \theta] \geq [1 + \frac{\partial}{\partial\theta}E(\theta_V^* - \theta \mid \theta)]^2\, [I(\theta)]^{-1} + [E(\theta_V^* \mid \theta) - \theta]^2 \ .$$

Note that this inequality holds for any estimator, θ_V^* , of θ .

V Second Modified Test Information Function

For the maximum likelihood estimate $\hat{\theta}_V$, we can rewrite the inequality (4.2) by using the MLE bias function, which is given by (3.4), to obtain

(5.1)
$$E[(\hat{\theta}_V - \theta)^2 \mid \theta] \geq [1 + \frac{\partial}{\partial\theta}B(\hat{\theta}_V \mid \theta)]^2\, [I(\theta)]^{-1} + [B(\hat{\theta}_V \mid \theta)]^2 \ .$$

Taking the reciprocal of the right hand side of (5.1), which is an approximate minimum bound of the mean squared error of the maximum likelihood estimator, the second modified test information function, $\Xi(\theta)$, is proposed by

$$(5.2) \qquad \Xi(\theta) = I(\theta)\ \{[1 + \frac{\partial}{\partial\theta}B(\hat{\theta}_V \mid \theta)]^2 + I(\theta)\ [B(\hat{\theta}_V \mid \theta)]^2\}^{-1} \ .$$

We can see that the difference between the two modification formuae of the test information function, which are defined by (3.7) and (5.2), respectively, is the second and last term in the braces of the right hand side of the formula (5.2). Since this term is nonnegative, there is a relationship

$$(5.3) \qquad \Xi(\theta) \leq \Upsilon(\theta) \ ,$$

throughout the whole range of θ , regardless of the slope of the MLE bias function. If there is a range of θ where the maximum likelihood estimate is unbiased, then we will have

$$(5.4) \qquad \Xi(\theta) = \Upsilon(\theta) = I(\theta) \ .$$

Since under a general condition the maximum likelihood estimator $\hat{\theta}_V$ is asymptotically unbiased as the number of items approaches positive infinity, (5.4) holds asymptotically for all θ .

VI Examples

Samejima has applied formula (3.5) for the MLE bias functions of the Iowa Level 11 Vocabulary Subtest and Shiba's Test J1 of Word/Phrase Comprehension, based upon the set of data collected for 2,356 and 2,259 subjects, respectively. These tests have forty-three and fifty-five dichotomously scored items, respectively, and following the normal ogive model, whose operating characteristic for the correct answer is given by

$$(6.1) \qquad P_g(\theta) = [2\pi]^{-1/2} \int_{-\infty}^{a_g(\theta - b_g)} e^{-u^2/2}\, du \ ,$$

the discrimination and difficulty parameters were estimated (Samejima, 1984a, 1984b). Tables 6-1 and 6-2 present those estimated item parameters. The resulting MLE bias functions are illustrated in Figure 6-1. We can see that in each of these two examples there is a wide range of θ , i.e., approximately (-2.0, 1.5), for which the maximum likelihood estimate of θ is practically unbiased. The amount of bias is especially small for Shiba's Test J1. Although this feature indicates good qualities of these tests, we still have to expect some biases when these tests are administered to groups of examinees whose ability distributes on the relatively lower side or on the relatively higher side of the ability scale.

When the MLE bias function of the test is monotone increasing, as are those illustrated in Figure 6-1, it is obvious from (3.7) that $\Upsilon(\theta)$ will assume lesser values than those of the original test information function $I(\theta)$ for lower and higher levels of θ , while these two functions are practically identical in between. The same applies to $\Xi(\theta)$, and we have the relationship,

$$(6.2) \qquad \Xi(\theta) \leq \Upsilon(\theta) \leq I(\theta) \ ,$$

throughout the whole range of θ .

Differentiating (6.1) twice with respect to θ and rearranging, we obtain

TABLE 6-1

Estimated Item Discrimination Parameter $\hat{a}_g$ and Item Difficulty Parameter $\hat{b}_g$ for Each of the Forty-Three Dichotomous Test Items of the Level 11 Vocabulary Subtest of the Iowa Tests of Basic Skills, Based upon the Results Collected for 2,356 School Children of Approximately Age Eleven.

Item g	Discrimination Parameter $\hat{a}_g$	Difficulty Parameter $\hat{b}_g$
24	0.196	-4.257
25	0.829	-1.000
26	0.614	-0.821
27	0.594	-0.340
28	0.669	-0.900
29	0.867	-1.077
30	0.956	-0.557
31	0.938	-0.179
32	0.940	-0.803
33	0.434	-2.331
34	0.598	-1.210
35	0.489	-0.569
36	0.657	-0.987
37	0.351	0.577
38	0.665	-0.468
39	0.333	-0.676
40	0.683	0.402
41	0.531	-0.948
42	0.436	0.258
43	0.672	-0.867
44	0.143	4.175
45	0.898	-0.357
46	0.612	-0.318
47	0.494	-0.781
48	0.849	0.054
49	0.421	-0.626
50	0.346	-0.250
51	0.664	-0.420
52	0.640	0.217
53	0.402	0.526
54	0.573	0.126
55	0.667	-0.342
56	0.593	1.007
57	0.370	0.398
58	0.416	0.782
59	0.491	-0.731
60	0.678	-0.170
61	0.519	0.748
62	0.938	-0.485
63	0.637	-0.398
64	0.818	-0.042
65	0.606	0.595
66	0.604	-0.376

TABLE 6-2

Estimated Item Discrimination Parameter $\hat{a}_g$ and Item Difficulty Parameter $\hat{b}_g$ for Each of the Fifty-Five Dichotomous Test Items of Test J1 of Shiba's Word/Phrase Comprehension Tests, Based upon the Results Collected for 2,259 Junior High School Students.

Item g	Discrimination Parameter $\hat{a}_g$	Difficulty Parameter $\hat{b}_g$
J101	0.726	-0.238
J102	0.537	-0.956
J103	0.568	-1.263
J104	0.710	-0.809
J105	0.794	-0.097
J106	0.495	-0.741
J107	0.583	0.205
J108	0.771	-1.974
J109	0.386	-0.872
J110	0.572	-0.327
J111	0.950	-1.266
J112	0.437	-1.036
J113	0.508	-1.061
J114	0.472	0.486
J115	0.704	-0.224
J116	0.303	-1.671
J117	0.390	-0.626
J118	0.583	-1.573
J119	0.653	-0.972
J120	0.293	1.058
J121	0.470	-0.904
J122	0.451	-1.038
J123	0.456	0.151
J124	0.562	-1.313
J125	0.450	-1.691
J126	0.367	-0.424
J127	0.525	-1.299
J128	0.679	-1.094

Item g	Discrimination Parameter $\hat{a}_g$	Difficulty Parameter $\hat{b}_g$
J129	0.761	1.416
J130	0.351	-1.839
J131	0.798	-0.494
J132	0.322	0.162
J133	0.822	-1.377
J134	0.302	1.633
J135	0.850	-0.225
J136	0.368	0.264
J137	0.591	0.331
J138	—	—
J139	0.375	1.602
J140	0.422	0.216
J141	0.566	-0.689
J142	0.447	0.132
J143	0.586	-0.100
J144	0.384	-0.399
J145	0.630	-0.479
J146	0.880	0.057
J147	0.333	0.374
J148	0.521	-0.062
J149	0.509	-0.108
J150	0.512	-0.040
J151	0.462	0.907
J152	0.394	0.478
J153	0.384	2.029
J154	0.242	2.353
J155	0.738	1.258
J156	0.655	1.468

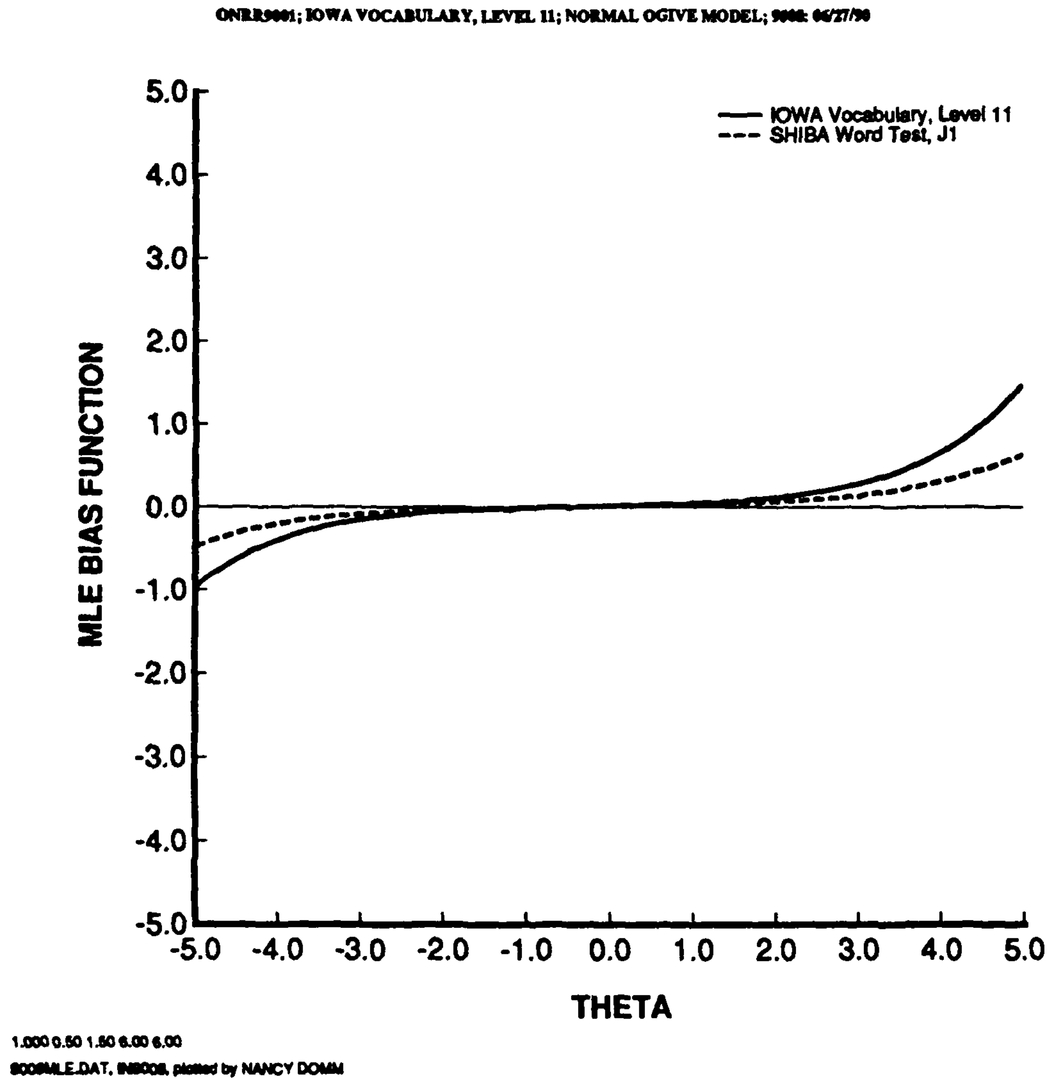


FIGURE 6-1

MLE Bias Functions of the Iowa Level 11 Vocabulary Subtest (Solid Line) and of Shiba's Test J1 of Word/Phrase Comprehension (Dashed Line), Following the Normal Ogive Model.

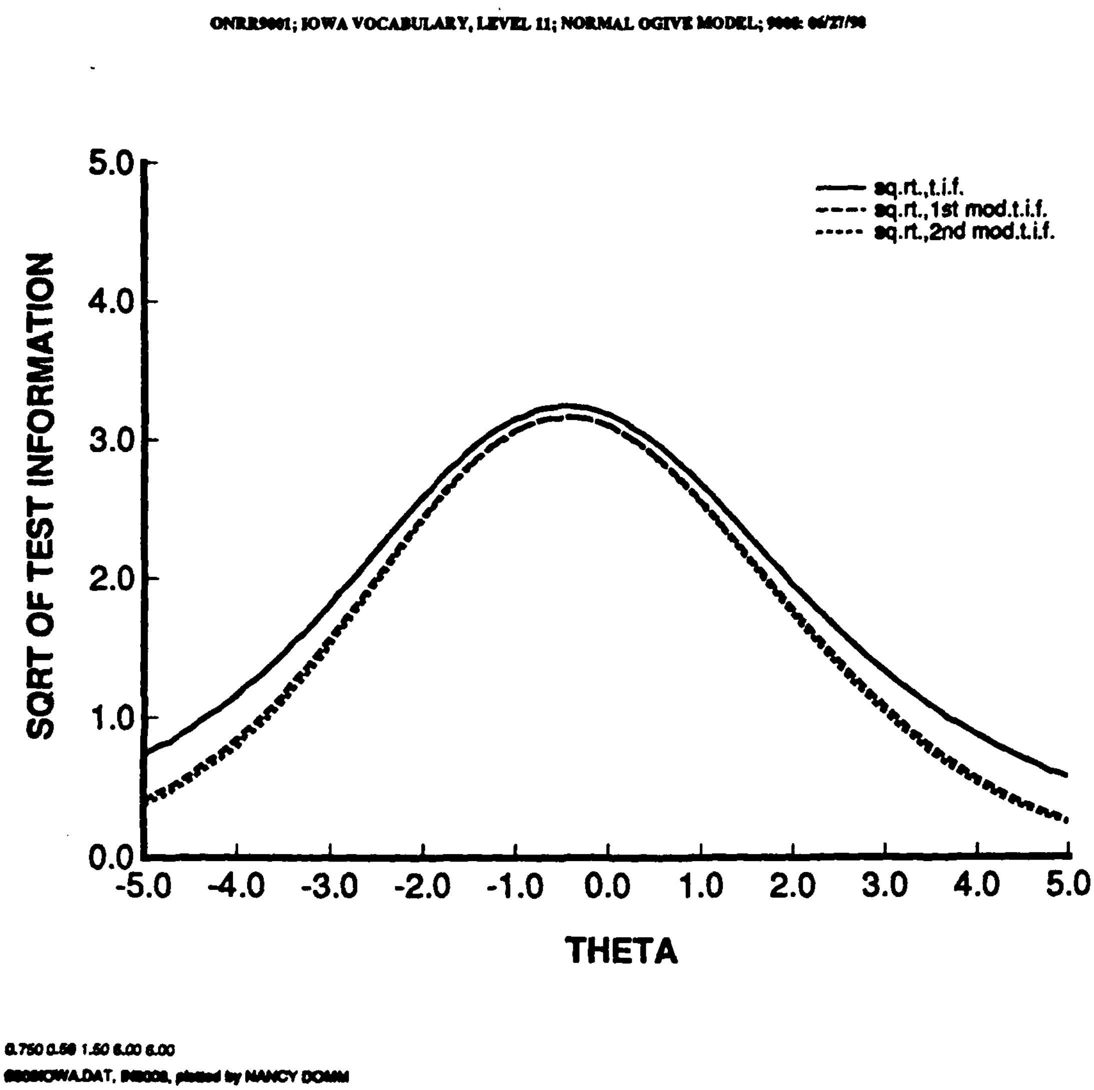
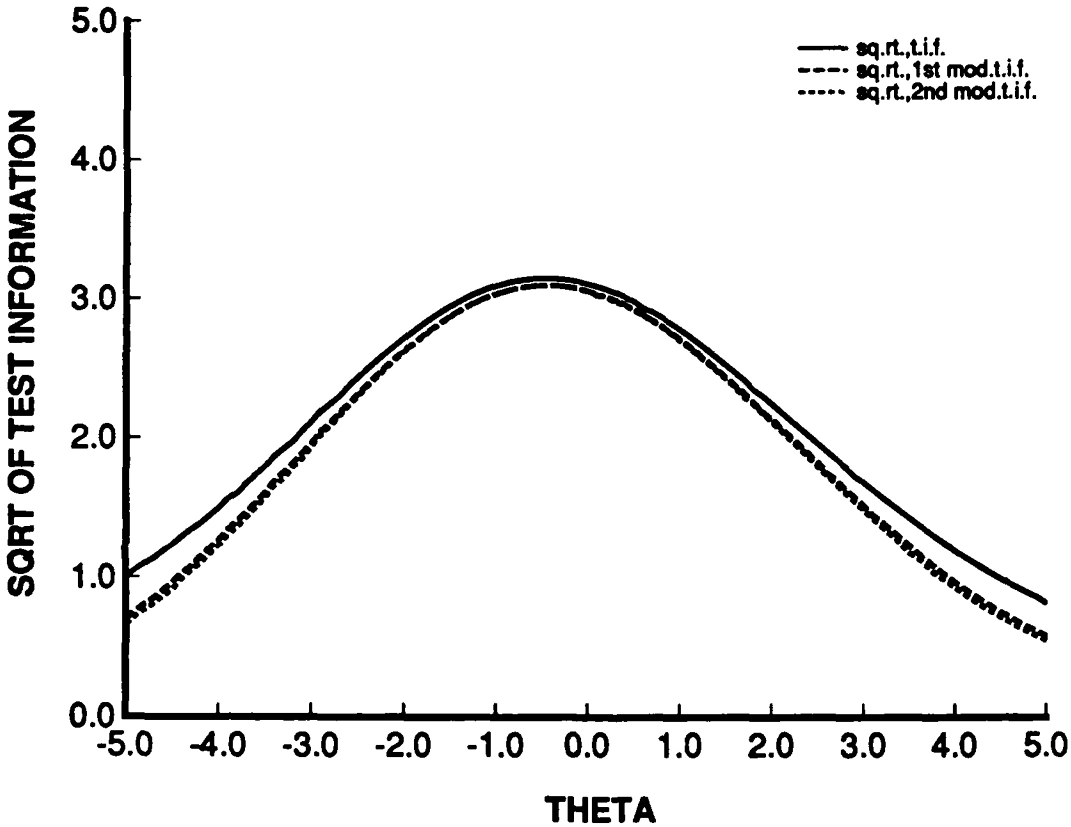


FIGURE 6-2

Square Roots of the Original (Solid Line) and the Two Modified (Dashed and Dotted Lines) Test Information Functions of the Iowa Level 11 Vocabulary Subtest, Following the Normal Ogive Model.

ONRR9001; SHIBA WORD TEST, J1; NORMAL OGIVE MODEL; 9008: 06/27/90



0.780 0.80 1.80 6.00 6.00
[illegible].DAT, [illegible], plotted by NANCY DOMM

FIGURE 6-3

Square Roots of the Original (Solid Line) and the Two Modified (Dashed and Dotted Lines) Test Information Functions of Shiba's Test J1 of Word/Phrase Comprehension, Following the Normal Ogive Model.

$$P_g'(\theta) = [2\pi]^{-1/2} a_g \exp[-(1/2)\, a_g^2(\theta - b_g)^2] \tag{6.3}$$

and

$$P_g''(\theta) = -a_g^2(\theta - b_g)\, P_g'(\theta) \ . \tag{6.4}$$

Substituting (6.3) and (6.4) into (3.5) and rearranging, we can write for the MLE bias function following the normal ogive model on the dichotomous response level

$$B(\hat{\theta}_V \mid \theta) = (1/2)\, [I(\theta)]^{-2} \sum_{g=1}^{n} a_g^2(\theta - b_g)\, I_g(\theta) \ . \tag{6.5}$$

Differentiating (6.5) with respect to θ , we obtain

$$\begin{aligned} \frac{\partial}{\partial\theta} B(\hat{\theta}_V \mid \theta) \quad &= \quad [I(\theta)]^{-2}[(1/2) \sum_{g=1}^{n} a_g^2\, [I_g'(\theta)(\theta - b_g) + I_g(\theta)] \\ &\quad -[I(\theta)]^{-1}\, I'(\theta) \sum_{g=1}^{n} a_g^2(\theta - b_g)\, I_g(\theta)] \ . \end{aligned} \tag{6.6}$$

It is obvious from (1.4), (1.8) and (6.6) that we have

$$I_g'(\theta) = I_g(\theta)\, [P_g'(\theta)\, \{2P_g(\theta) - 1\}\, (P_g(\theta)\{1 - P_g(\theta)\})^{-1} - 2a_g^2(\theta - b_g)] \tag{6.7}$$

and

$$I'(\theta) = \sum_{g=1}^{n} I_g(\theta)\, [P_g'(\theta)\, \{2P_g(\theta) - 1\}\, (P_g(\theta)\{1 - P_g(\theta)\})^{-1} - 2a_g^2(\theta - b_g)] \ . \tag{6.8}$$

Figures 6-2 and 6-3 show the square roots of the original and the two modified test information functions for the Iowa Level 11 Vocabulary Subtest and Shiba's Test J1 of Word/Phrase Comprehension, respectively, following the normal ogive model. In each of these figures, the curves respresenting the results of the two modification formulae assume lower values than the square root of the original test information function for all θ , as was expected from the shape of the MLE bias function in Figure 6-1. The discrepancies between the results of the two modification formulae are small, however, in each figure.

In the three-parameter logistic model, the operating characteristic of the correct answer is given by the formula (3.1), and Lord's MLE bias function for the three-parameter logistic model, which is given by (3.2), is readily applicable. Differentiating (3.1) three times with respect to θ and rearranging, we can write

$$P_g'(\theta) \ = (1 - c_g)\, Da_g\, \psi_g(\theta)\, [1 - \psi_g(\theta)] \ , \tag{6.9}$$

$$P_g''(\theta) \ = (1 - c_g)\, D^2 a_g^2\, \psi_g(\theta)\, [1 - \psi_g(\theta)]\, [1 - 2\psi_g(\theta)] = Da_g\, P_g'(\theta)[1 - 2\psi_g(\theta)] \tag{6.10}$$

and

(6.11) $$P_g'''(\theta) = D^2 a_g^2\ P_g'(\theta)[1 - 6\psi_g(\theta) + 6\{\psi_g(\theta)\}^2]\ ,$$

where $\psi_g(\theta)$ is defined by (3.3). Substituting (6.9) into (1.4) and rearranging, we obtain for the item information function

(6.12) $$I_g(\theta)\ = (1 - c_g)\ D^2 a_g^2\ \{\psi_g(\theta)\}^2\ [1 - \psi_g(\theta)]\ [c_g + (1 - c_g)\ \psi(\theta)]^{-1}\ .$$

This and (1.8) will enable us to evaluate Lord's MLE bias function given by (3.2). Differentiating (3.2) with respect to θ and rearranging, we can write

(6.13) $$\begin{aligned}\frac{\partial}{\partial\theta} B(\hat{\theta}_V \mid \theta) \ &= \ D\ \{I(\theta)\}^{-2}[\sum_{g=1}^{n} a_g\ I_g'(\theta)\{\psi_g(\theta) - (1/2)\} \\ &\quad + D \sum_{g=1}^{n} a_g^2\ I_g(\theta)\ \psi_g(\theta)\{1 - \psi_g(\theta)\} \\ &\quad - 2\ I'(\theta)\ \{I(\theta)\}^{-1} \sum_{g=1}^{n} a_g\ I_g(\theta)\ \{\psi_g(\theta) - (1/2)\}]\ .\end{aligned}$$

From (1.4), (3.1) and (1.8) we obtain for the first derivatives of the item and the test information functions with respect to θ

(6.14) $$\begin{aligned}I_g'(\theta) \ &= \ (1 - c_g)\ D^3 a_g^3\ \{\psi_g(\theta)\}^2\ [1 - \psi_g(\theta)]\ \{P_g(\theta)\}^{-1} \\ &\qquad [2 - 3\psi_g(\theta) - (1 - c_g)\ \psi_g(\theta)\{1 - \psi_g(\theta)\}\{P_g(\theta)\}^{-1}] \\ &= \ D a_g\ I_g(\theta)\ [2\{1 - \psi_g(\theta)\} - \psi_g(\theta)\{P_g(\theta)\}^{-1}]\end{aligned}$$

and

(6.15) $$I'(\theta) = D \sum_{g=1}^{n} a_g\ I_g(\theta)\ [2\{1 - \psi_g(\theta)\} - \psi_g(\theta)\{P_g(\theta)\}^{-1}]\ ,$$

and we can use these two results in (6.13) in order to evaluate $\frac{\partial}{\partial\theta} B(\hat{\theta}_V \mid \theta)$.

When $c_g = 0$, i.e., for the original logistic model on the dichotomous response level, these formulae become much more simplified, and we can write

(6.16) $$P_g(\theta) \ = \ [1 + \exp\{-D a_g(\theta - b_g)\}]^{-1} \ = \psi_g(\theta)\ ,$$

(6.17) $$P_g'(\theta) \ = D a_g\ \psi_g(\theta)\ [1 - \psi_g(\theta)]\ ,$$

(6.18) $$P_g''(\theta) \ = D^2 a_g^2\ \psi_g(\theta)\ [1 - \psi_g(\theta)]\ [1 - 2\psi_g(\theta)] = D a_g\ P_g'(\theta)[1 - 2\psi_g(\theta)]\ ,$$

(6.19) $$P_g'''(\theta) \ = D^3 a_g^3\ \psi_g(\theta)\ [1 - \psi_g(\theta)]\ [1 - 6\psi_g(\theta) + 6\{\psi_g(\theta)\}^2]\ ,$$

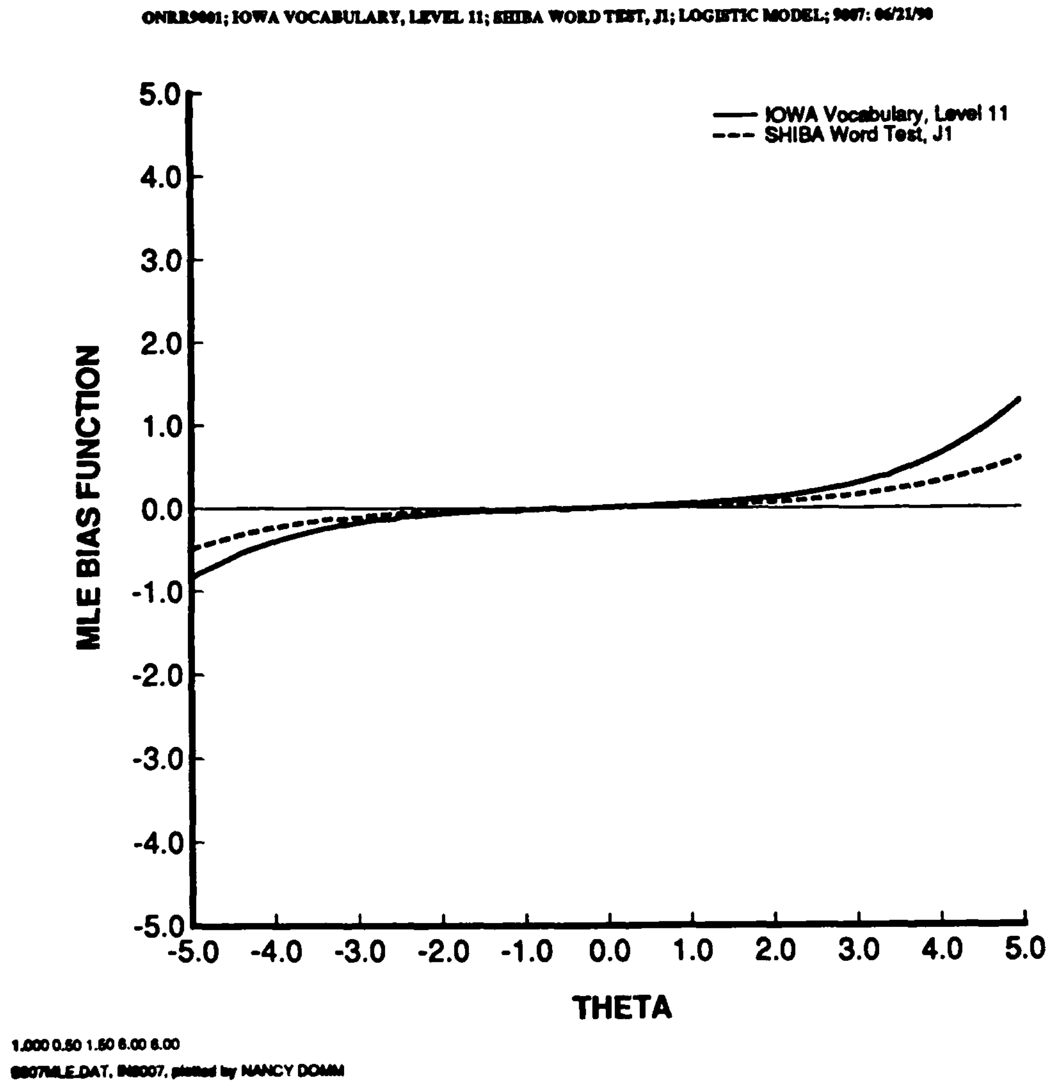
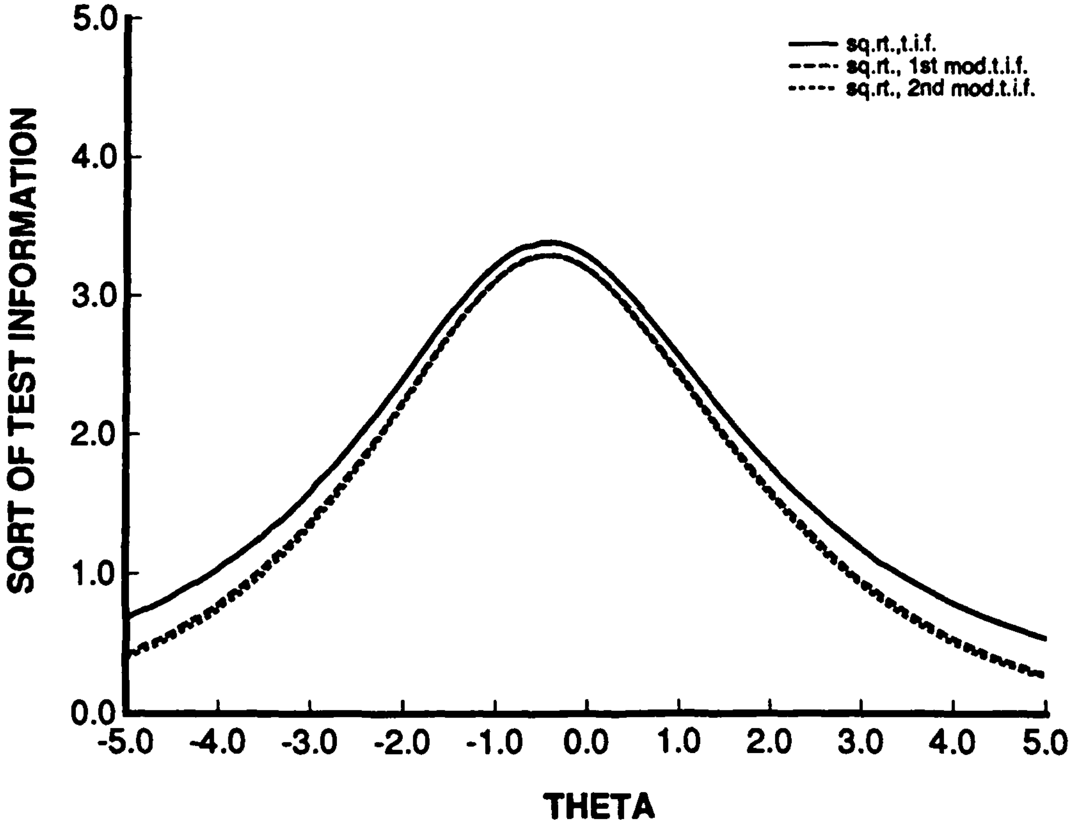


FIGURE 6-4

MLE Bias Functions of the Iowa Level 11 Vocabulary Subtest (Solid Line) and of Shiba's Test J1 of Word/Phrase Comprehension (Dashed Line), Following the Logistic Model.

ONRR9001; IOWA VOCABULARY, LEVEL 11; LOGISTIC MODEL; 9007: 06/21/90

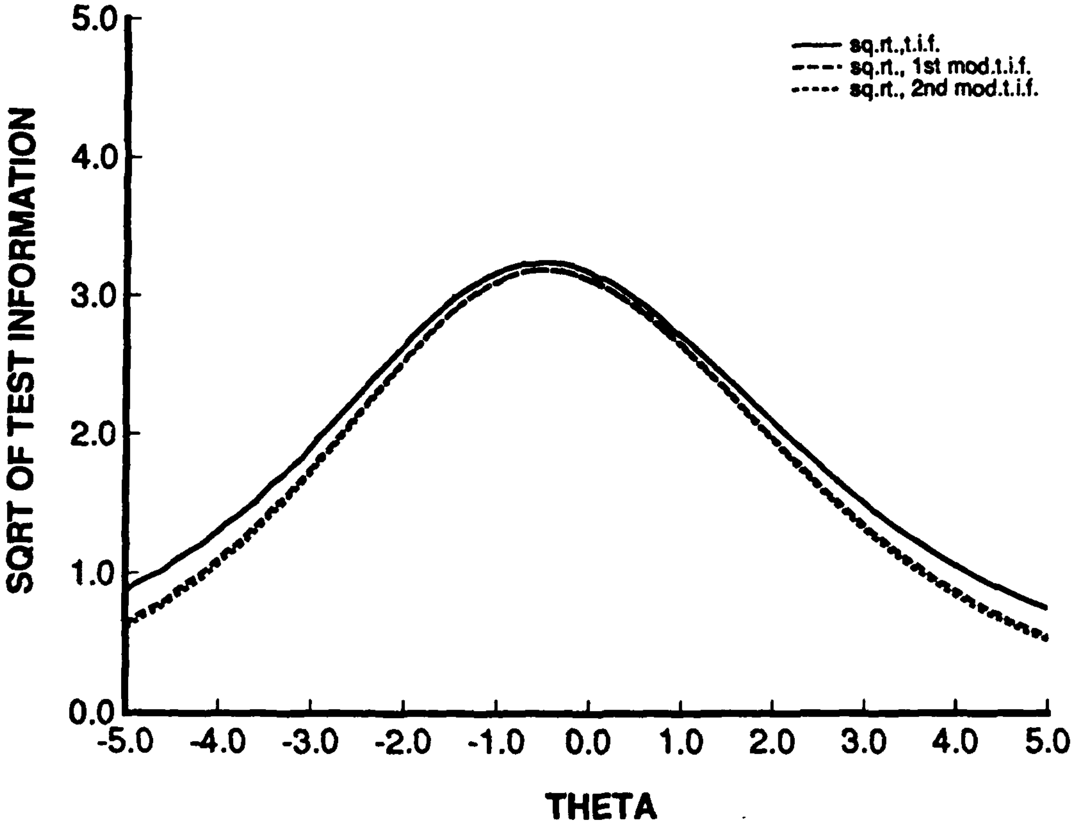


0.780 0.80 1.80 6.00 6.00
9007NOL.DAT, IN9007, plotted by NANCY DOMM

FIGURE 6-5

Square Roots of the Original (Solid Line) and the Two Modified (Dashed and Dotted Lines) Test Information Functions of the Iowa Level 11 Vocabulary Subtest, Following the Logistic Model.

ONRR9001; SHIBA WORD TEST, J1; LOGISTIC MODEL; 9007: 06/21/90



0.750 0.50 1.50 8.00 6.00
9007NDSL.DAT, 900007, plotted by NANCY DOMM

FIGURE 6-6

Square Roots of the Original (Solid Line) and the Two Modified (Dashed and Dotted Lines) Test Information Functions of Shiba's Test J1 of Word/Phrase Comprehension, Following the Logistic Model.

$$(6.20) \qquad I_g(\theta) = D^2 a_g^2 \, \psi_g(\theta) \, [1 - \psi_g(\theta)] \ ,$$

$$(6.21) \qquad I_g'(\theta) = D^3 a_g^3 \, \psi_g(\theta)[1 - \psi_g(\theta)][1 - 2\psi_g(\theta)] = D \, a_g \, I_g(\theta) \, [1 - 2\psi_g(\theta)] \ ,$$

$$(6.22) \qquad I(\theta) = D^2 \sum_{g=1}^{n} a_g^2 \, \psi_g(\theta) \, [1 - \psi_g(\theta)]$$

and

$$(6.23) \qquad I'(\theta) = D \sum_{g=1}^{n} a_g \, I_g(\theta) \, [1 - 2\psi_g(\theta)] \ ,$$

respectively. Thus the two modified test information functions, $\Upsilon(\theta)$ and $\Xi(\theta)$, which are defined by (3.7) and (5.2), respectively, can be evaluated accodingly, both for the original logistic model and for the three-parameter logistic model.

Figures 6-4 through 6-6 present the MLE bias functions and the square roots of the original and the two modified test information functions for the Iowa Level 11 Vocabulary Subtest and Shiba's Test J1 of Word/Phrase Comprehension, respectively, following the logistic model by using the same sets of estimated item parameters shown in Tables 6-1 and 6-2, and setting $D = 1.7$. These results are similar to those following the normal ogive model, which are presented by Figures 6-1 through 6-3, except that the square roots of the original and the modified test information functions are a little steeper, the characteristic of the logistic model in comparison with the normal ogive model.

In the homogeneous case of the graded response level (Samejima, 1969, 1972), the general formula for the operating characteristic of the item score $x_g \ (= 0, 1, ..., m_g)$ is given by

$$(6.24) \qquad P_{x_g}(\theta) = P^*_{x_g}(\theta) - P^*_{x_g+1}(\theta) \ ,$$

where

$$(6.25) \qquad P^*_{x_g}(\theta) = \int_{-\infty}^{a_g(\theta - b_{x_g})} \phi_g(t) \, dt \ ,$$

$$(6.26) \qquad -\infty = b_0 < b_1 < b_2 < ... < b_{m_g} < b_{m_g+1} = \infty \ ,$$

and $\phi_g(t)$ is some specified density function. When we replace the right hand side of (6.25) by that of (6.1) with b_g replaced by b_{x_g} and use the result in (6.24), we have the operating characteristic of x_g in the normal ogive model on the graded response level; when we do the same thing using the right hand side of (3.3), we obtain the operating characteristic of x_g in the logistic model on the graded response level.

Table 6-3 presents the item discrimination parameter a_g and the two item difficulty parameters, b_{x_g} for $x_g = 1, 2$, for each of the thirty-five hypothetical graded items. This hypothetical test gives an approximately constant amount of test information for the interval of θ , (-3, 3). Figure 6-7 presents the MLE bias function for this hypothetical test, following the normal ogive model and the logistic model on the graded response level, respectively. We can see that a practical unbiasedness holds for a very wide range of θ in both cases, as is expected for a set of graded test items whose response difficulty levels are widely distributed, an advantage of the graded response item over the dichotomous response item. We also notice that these two MLE bias functions are almost indistinguishable from

TABLE 6-3

Item Discrimination Parameter a_g and Two Item Difficulty Parameters b_{x_g}, $x_g = 1, 2$ for Each of the Thirty-Five Graded Test Items of a Hypothetical Test.

Item g	a_g	b_1	b_2
1	1.8	-4.75	-3.75
2	1.9	-4.50	-3.50
3	2.0	-4.25	-3.25
4	1.5	-4.00	-3.00
5	1.6	-3.75	-2.75
6	1.4	-3.50	-2.50
7	1.9	-3.00	-2.00
8	1.8	-3.00	-2.00
9	1.6	-2.75	-1.75
10	2.0	-2.50	-1.50
11	1.5	-2.25	-1.25
12	1.7	-2.00	-1.00
13	1.5	-1.75	-0.75
14	1.4	-1.50	-0.50
15	2.0	-1.25	-0.25
16	1.6	-1.00	0.00
17	1.8	-0.75	0.25
18	1.7	-0.50	0.50

Item g	a_g	b_1	b_2
19	1.9	-0.25	0.75
20	1.7	0.00	1.00
21	1.5	0.25	1.25
22	1.8	0.50	1.50
23	1.4	0.75	1.75
24	1.9	1.00	2.00
25	2.0	1.25	2.25
26	1.6	1.50	2.50
27	1.7	1.75	2.75
28	1.4	2.00	3.00
29	1.9	2.25	3.25
30	1.6	2.50	3.50
31	1.5	2.75	3.75
32	1.7	3.00	4.00
33	1.8	3.25	4.25
34	2.0	3.50	4.50
35	1.4	3.75	4.75

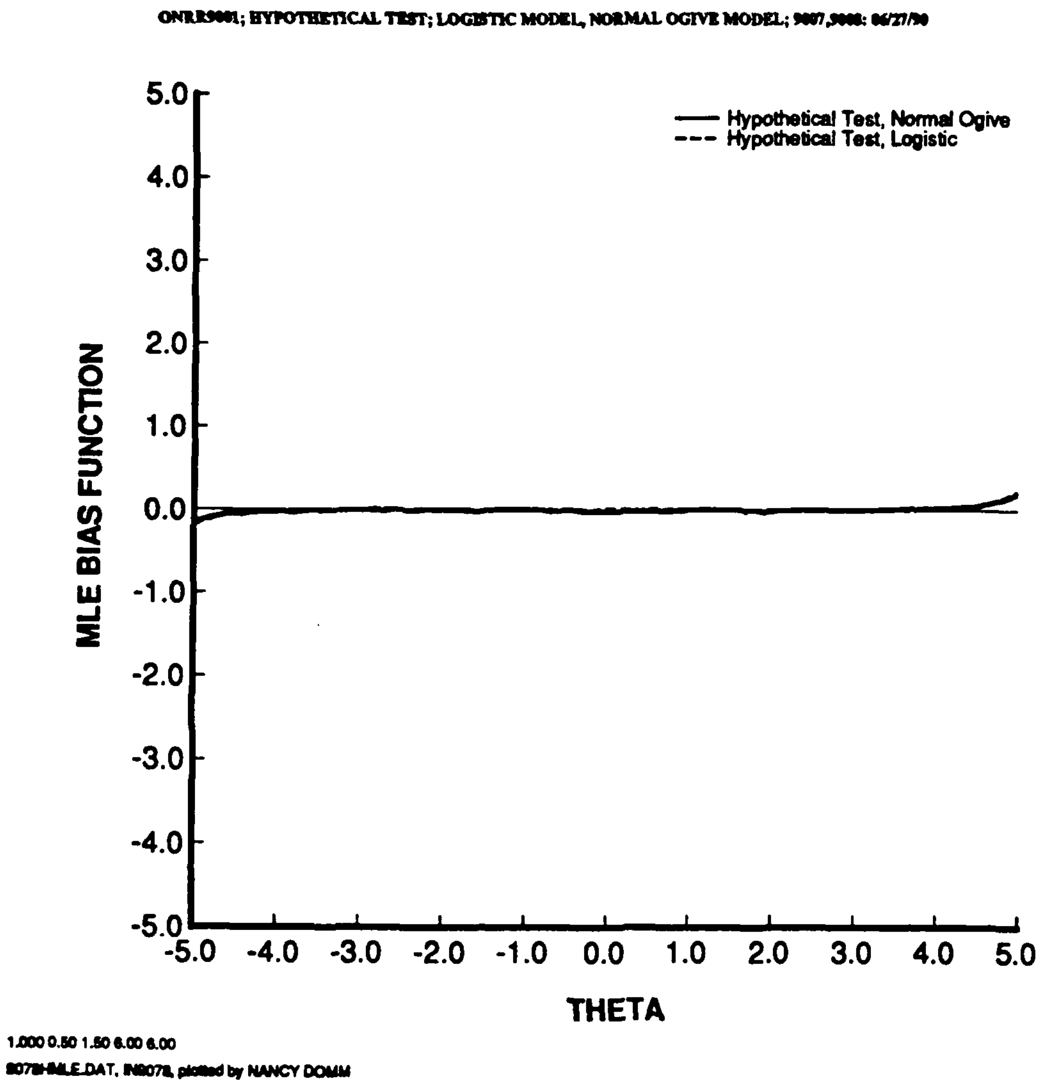
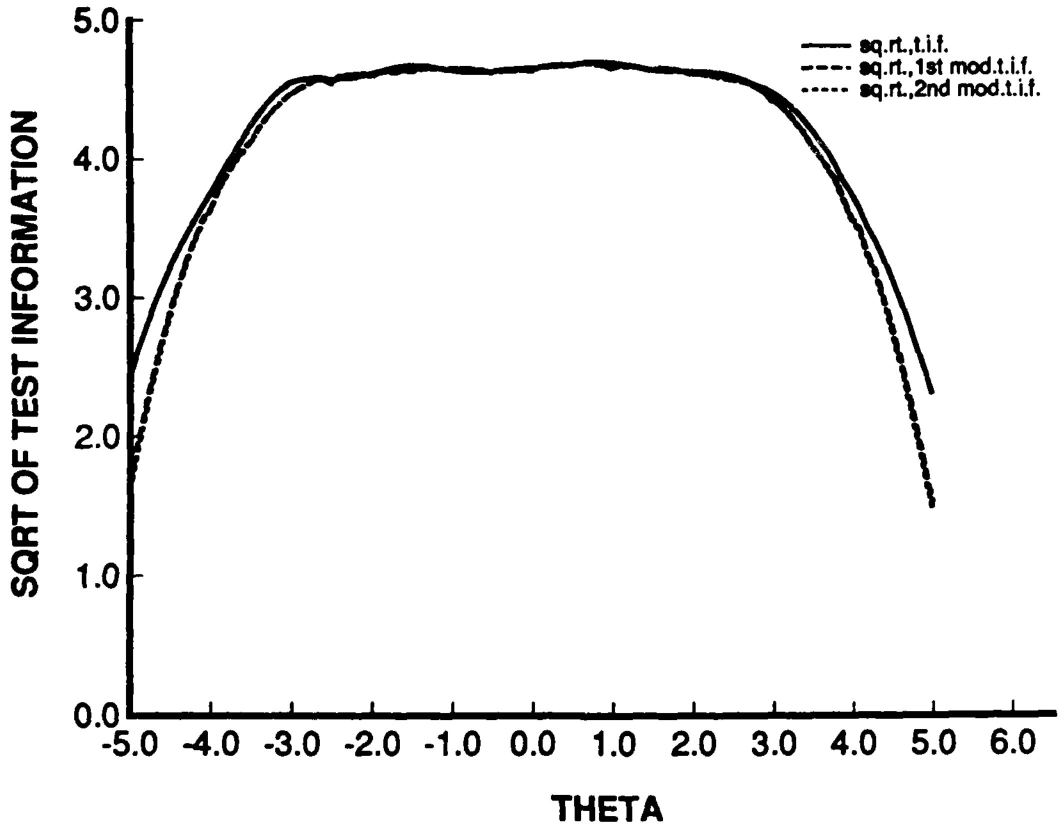


FIGURE 6-7

MLE Bias Functions of the Hypothetical Test of Thirty-Five Graded Test Items Following the Normal Ogive Model (Solid Line) and the Logistic Model (Dashed Line).

ONRR9001; HYPOTHETICAL TEST; NORMAL OGIVE MODEL; 9008: 06/27/90

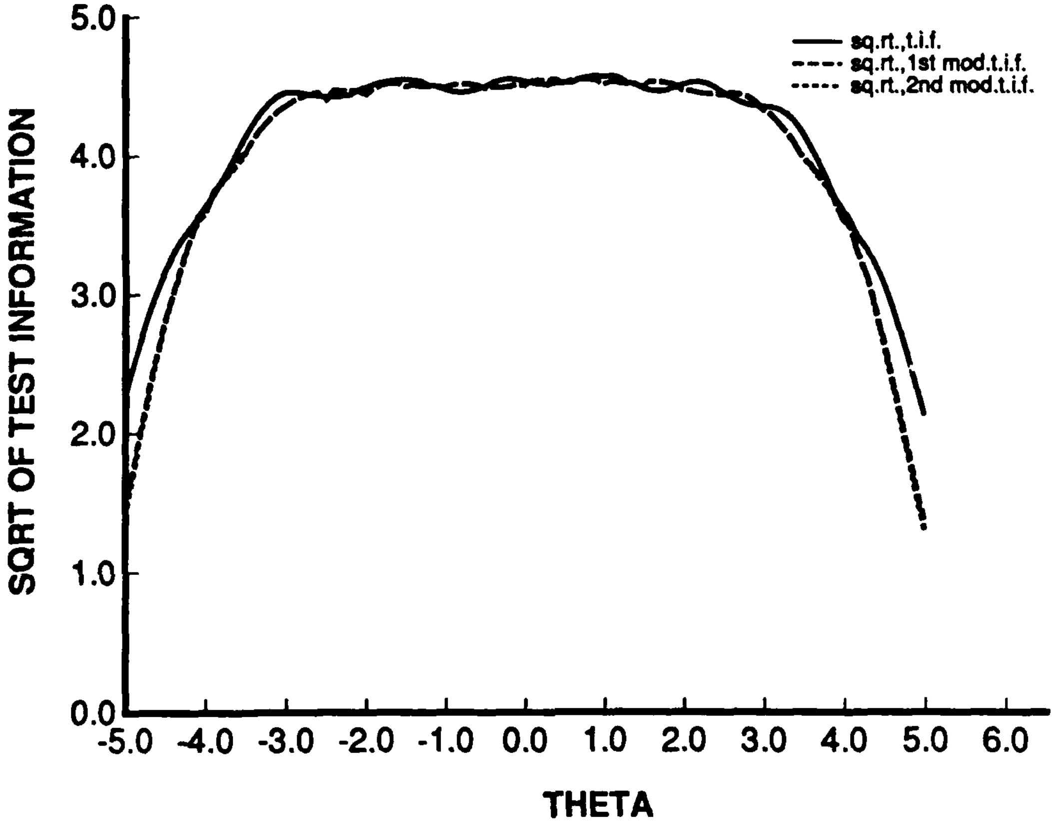


0.750 0.50 1.60 6.00 6.00
[illegible]HYPO.DAT, [illegible], plotted by NANCY DOMM

FIGURE 6-8

Square Roots of the Original (Solid Line) and the Two Modified (Dashed and Dotted Lines) Test Information Functions of the Hypothetical Test of Thirty-Five Graded Test Items Following the Normal Ogive Model.

ONRR9001; HYPOTHETICAL TEST; LOGISTIC MODEL; 9007: 06/27/90



0.750 0.50 1.50 6.00 6.00

[illegible]HYPO.DAT, [illegible], plotted by NANCY DOMM

FIGURE 6-9

Square Roots of the Original (Solid Line) and the Two Modified (Dashed and Dotted Lines) Test Information Functions of the Hypothetical Test of Thirty-Five Graded Test Items Following the Logistic Model.

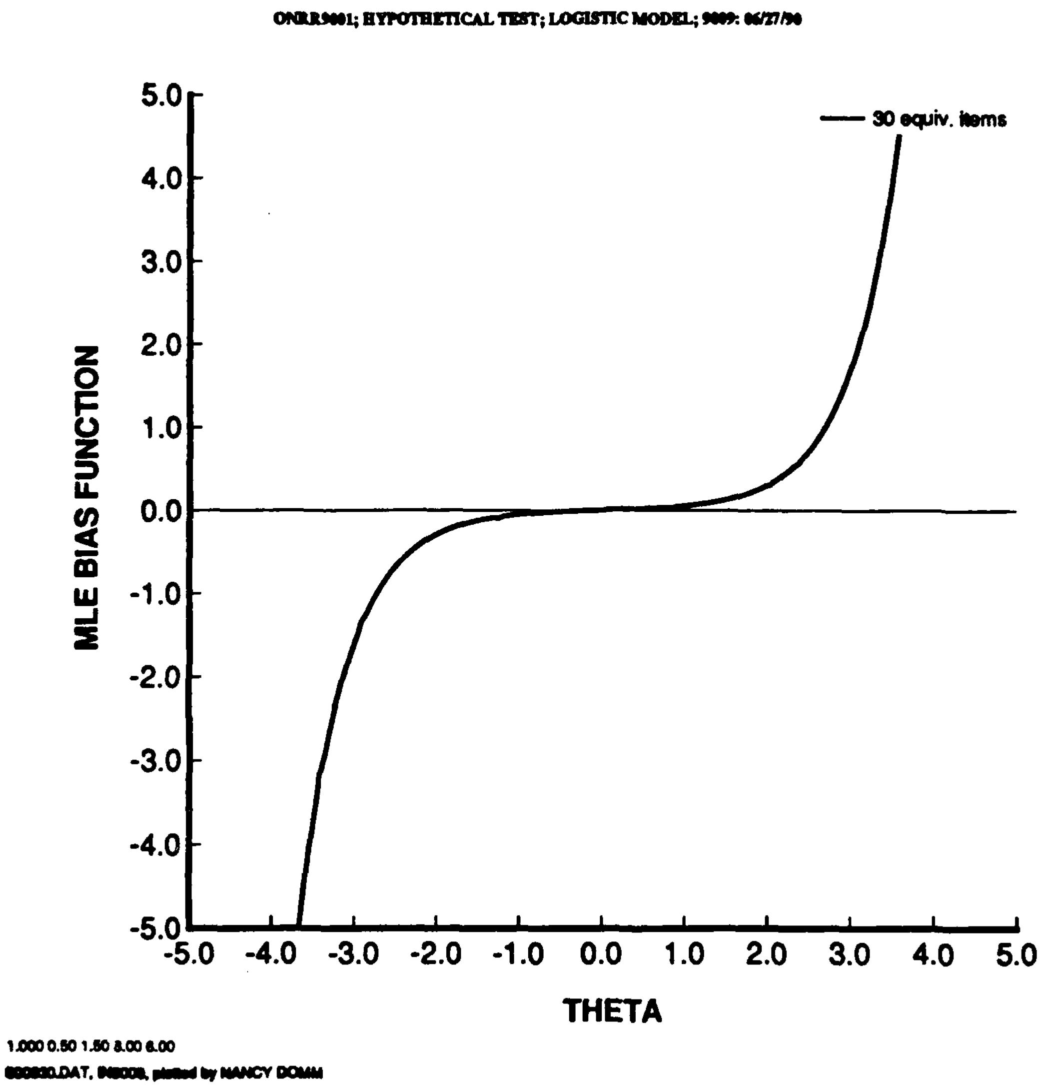
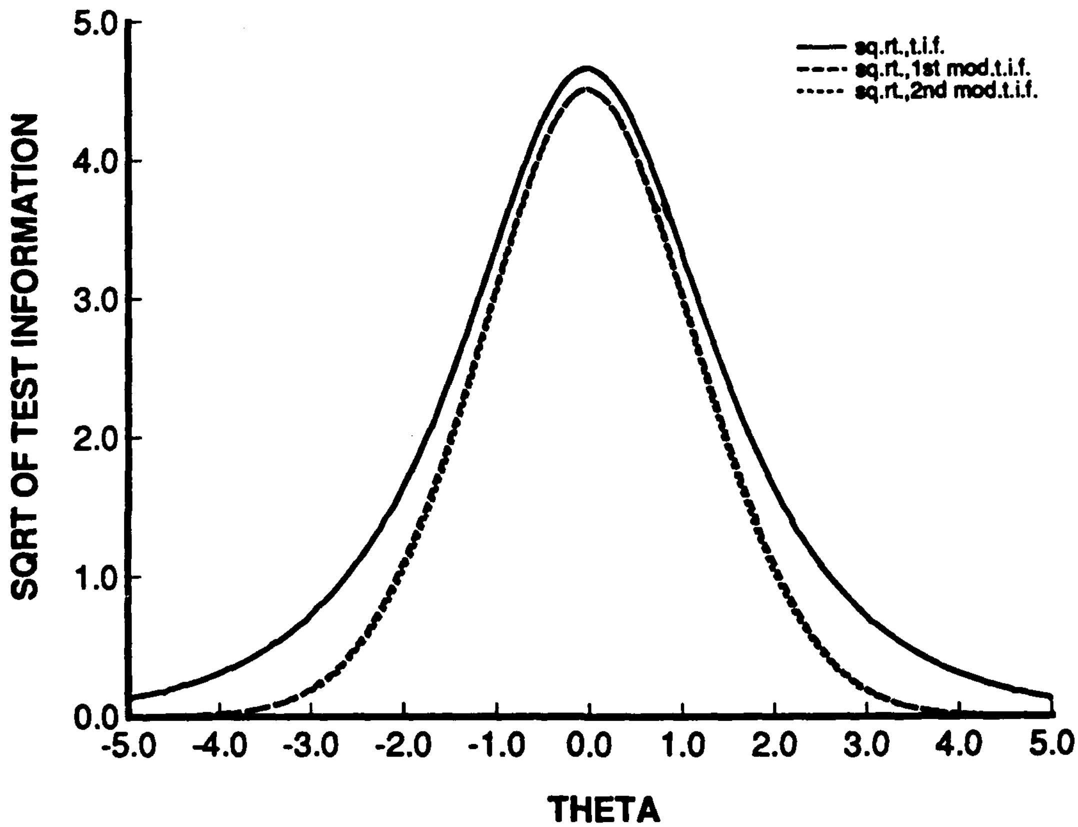


FIGURE 6-10

MLE Bias Function of the Hypothetical Test of Thirty Equivalent Test Items Following the Logistic Model with $a_g = 1.0$ and $b_g = 0.0$ As the Common Parameters.

ONRR9001; HYPOTHETICAL TEST; LOGISTIC MODEL; 9009: 06/27/90



0.750 0.50 1.50 6.00 6.00

[illegible].DAT, [illegible], plotted by NANCY DOMM

FIGURE 6-11

Square Roots of the Original (Solid Line) and the Two Modified (Dashed and Dotted Lines) Test Information Functions of the Hypothetical Test of Thirty Equivalent Items Following the Logistic Model with $a_g = 1.0$ and $b_g = 0.0$ As the Common Parameters.

each other. Figures 6-8 and 6-9 present the square roots of the original and the two modified test information functions of this hypothetical test of graded items, following the normal ogive model and the logistic model, respectively. As is expected, the differences among the three functions are small for a wide range of θ in both cases. It is interesting to note, however, that in these figures the square roots of the modified test information functions assume higher values than the square root of the original test information function at certain points of θ, and this tendency is especially conspicuous in the results of the logistic model. This comes from the fact that the MLE bias functions, which are presented in Figure 6-7 for both models, have tiny ups and downs, and they are not strictly increasing in θ.

We notice that in each of the examples given above, the difficulty parameters of these items in each test distribute widely over the range of θ of interest, as we can see in Tables 3-1 through 3-3. This fact is the main reason that the MLE bias function assumes relatively small values for a wide range of θ, and that the resulting two modified test information functions are reasonably close to the original test information function. For the sake of comparison, Figures 6-10 and 6-11 present the MLE bias function and the square roots of the original and the two modified test information functions, respectively, for a hypothetical test of thirty equivalent items with the common item parameters, $a_g = 1.0$ and $b_g = 0.0$, following the logistic model. We can see in Figure 6-10 that the amount of bias increases rapidly outside the range of θ, $(-1.0, 1.0)$. The resulting square roots of the two modified test information functions demonstrate substantially large decrements from the original $[I(\theta)]^{1/2}$ outside this interval of θ, as we can see in Figure 6-11.

We also notice that in all these examples there are not substantial differences between the results of the two modification formulae. This indicates that in these examples it does not make so much difference if we choose Modification Formula No. 1 or Modification Formula No. 2. We should not generalize this conclusion to other situations, however, until we have tried these modification formulae on different types of data sets.

VII Effect of the Modifications of the Test Information Function in Efficiency in Computerized Adaptive Testing

Amount of test information can be used effectively in the stopping rule of the computerized adaptive testing. A procedure may be to terminate the presentation of a new item out of the itempool to the individual examinee when $I(\theta)$ has reached an a priori set amount at the current value of his estimated θ.

We notice that for the stopping rule in computerised adaptive testing the modified test informations will serve better, for in many cases our itempool is limited, and especially for examinees whose ability levels are close to the upper or the lower end of the configuration of the difficulty parameters of the items in the itempool there are not so many optimal items. In such a case, even if the amount ot test information has reached a certain criterion level it does not mean that their ability levels are estimated with the same accuracy as those of individuals of intermediate ability levels, as was observed in Section 1. Since, taking the MLE bias function into consideration, the two modified test information functions, $\Upsilon(\theta)$ and $\Xi(\theta)$, are based upon a more meaningful minimum bound of the conditional variance and upon a minimum bound of the mean squared error of the maximum likelihood estimator, respectively, they will be effectively used as stopping rules in computerised adaptive testing, especially for individuals of lower and higher ends of ability levels.

Since the test information function $I(\theta)$ and its two modification formulae, $\Upsilon(\theta)$ and $\Xi(\theta)$, are likely to be the ones exemplified in Figure 6-11 in the process of adaptive testing, provided that the program for the test is written well, we should expect visible differences between the results obtained by using $I(\theta)$ and one of its modification formulae, especially for subjects whose ability levels are close to the upper or lower end of the ability interval of interest.

This is one topic we need to investigate in the future, specifying the amount of improvement with simulated and empirical data collected in computerised adaptive testing.

VIII Minimum Bounds of Variance and Mean Squared Error for the Transformed Latent Variable

Since most psychological scales, including those in latent trait models, are subject to *monotone* transformation, we need to consider information functions that are based upon the transfomed latent variable. Let τ denote a transformed latent variable, i.e.,

$$\tau = \tau(\theta) \ . \tag{8.1}$$

We assume that τ is strictly increasing in, and three times differentiable with respect to, θ , and vice versa. We have for the operating characteristic, $P^*_{k_g}(\tau)$, of the discrete item response k_g , which is defined as a function of τ ,

$$P^*_{k_g}(\tau) = prob.[k_g \mid \tau] = prob.[k_g \mid \theta] = P_{k_g}(\theta) \ , \tag{8.2}$$

and by local independence we can write for the operating characteristic of the response pattern, $P^*_V(\tau)$,

$$P^*_V(\tau) = \prod_{k_g \epsilon V} P^*_{k_g}(\tau) = \prod_{k_g \epsilon V} P_{k_g}(\theta) = P_V(\theta) \ . \tag{8.3}$$

As before, the item response information function, $I^*_{k_g}(\tau)$, is defined by

$$I^*_{k_g}(\tau) = -\frac{\partial^2}{\partial \tau^2} \log P^*_{k_g}(\tau) \ , \tag{8.4}$$

and for the item information function, $I^*_g(\tau)$, and the test information function, $I^*(\tau)$, we can write from (8.4), (1.3) and (1.8)

$$\begin{aligned} I^*_g(\tau) &= \sum_{k_g} I^*_{k_g}(\tau)\, P^*_{k_g}(\tau) = \sum_{k_g} [\frac{\partial}{\partial \tau} P^*_{k_g}(\tau)]^2\, [P^*_{k_g}(\tau)]^{-1} \\ &= \sum_{k_g} [\frac{\partial}{\partial \theta} P_{k_g}(\theta)\, \frac{\partial \theta}{\partial \tau}]^2\, [P_{k_g}(\theta)]^{-1} = I_g(\theta)\, [\frac{\partial \theta}{\partial \tau}]^2 \end{aligned} \tag{8.5}$$

and

$$I^*(\tau) = \sum_{g=1}^{n} I^*_g(\tau) = I(\theta)\, [\frac{\partial \theta}{\partial \tau}]^2 \ , \tag{8.6}$$

respectively. Let τ^*_V be any estimator of τ , which may be biased or unbiased. In general, we can write

$$E(\tau^*_V \mid \tau) = \tau + E(\tau^*_V - \tau \mid \tau) \ , \tag{8.7}$$

and, differentiating (8.7) with respect to θ , we obtain

$$(8.8) \qquad \frac{\partial}{\partial\theta}E(\tau_V^* \mid \tau) = \frac{\partial\tau}{\partial\theta} + \frac{\partial}{\partial\theta}E(\tau_V^* - \tau \mid \tau) \ .$$

Since from (8.3) we can also write for $E(\tau_V^* \mid \tau)$

$$(8.9) \qquad E(\tau_V^* \mid \tau) = \sum_V \tau_V^* \ P_V^*(\tau) = \sum_V \tau_V^* \ P_V(\theta) \ ,$$

differentiating (8.9) with respect to θ and following a logic similar to that used in Section 2, we obtain

$$(8.10) \qquad \begin{aligned} \frac{\partial}{\partial\theta}E(\tau_V^* \mid \tau) &= \frac{\partial}{\partial\theta}\sum_V \tau_V^* \ P_V(\theta) = \sum_V [\tau_V^* - E(\tau_V^* \mid \tau)] \ [\frac{\partial}{\partial\theta}P_V(\theta)] \\ &= \sum_V [\tau_V^* - E(\tau_V^* \mid \tau)] \ [\frac{\partial}{\partial\theta}\log P_V(\theta)] \ P_V(\theta) \ . \end{aligned}$$

By the Cramér-Rao inequality, we can write

$$(8.11) \qquad [\frac{\partial}{\partial\theta}E(\tau_V^* \mid \tau)]^2 \leq Var.(\tau_V^* \mid \tau) \ E[\{\frac{\partial}{\partial\theta}\log P_V(\theta)\}^2] \ ,$$

and from this, (1.7), (1.8), (2.10) and (8.8) we obtain

$$(8.12) \qquad \begin{aligned} Var.(\tau_V^* \mid \tau) &\geq [\frac{\partial}{\partial\theta}E(\tau_V^* \mid \tau)]^2 \ [I(\theta)]^{-1} \\ &= [\frac{\partial\tau}{\partial\theta} + \frac{\partial}{\partial\theta}E(\tau_V^* - \tau \mid \tau)]^2 \ [I(\theta)]^{-1} \ . \end{aligned}$$

Thus the rightest hand side of (8.12) provides us with the minimum variance bound of any estimator of τ. When τ_V^* is an unbiased estimator of τ, the second term of the first factor of the rightest hand side of (8.12) equals zero, and by virtue of (8.6) the inequality is reduced to

$$(8.13) \qquad Var.(\tau_V^* \mid \tau) \geq [\frac{\partial\tau}{\partial\theta}]^2 \ [I(\theta)]^{-1} = [I^*(\tau)]^{-1} \ .$$

For the mean squared error, $E[(\tau_V^* - \tau)^2 \mid \tau]$, we can write

$$(8.14) \qquad E[(\tau_V^* - \tau)^2 \mid \tau] = Var.(\tau_V^* \mid \tau) + [E(\tau_V^* \mid \tau) - \tau]^2 \ ,$$

and from this and (8.12) we obtain

$$(8.15) \qquad E[(\tau_V^* - \tau)^2 \mid \tau] \geq [\frac{\partial\tau}{\partial\theta} + \frac{\partial}{\partial\theta}E(\tau_V^* - \tau \mid \tau)]^2 \ [I(\theta)]^{-1} + [E(\tau_V^* \mid \tau) - \tau]^2 \ .$$

IX Modified Test Information Functions Based upon the Transformed Latent Variable

The maximum likelihood estimator, $\hat{\tau}_V$, of τ , can be obtained by the direct transformation of the maximum likelihood estimate, $\hat{\theta}_V$, of θ , i.e.,

$$\hat{\tau}_V = \tau(\hat{\theta}_V) \ . \tag{9.1}$$

Let $B^*(\hat{\tau}_V \mid \tau)$ be the MLE bias function defined for the transformed latent variable τ , i.e.,

$$B^*(\hat{\tau}_V \mid \tau) = E(\hat{\tau}_V - \tau \mid \tau) \ . \tag{9.2}$$

From this, (8.12) and (8.15) we obtain

$$Var.(\hat{\tau}_V \mid \tau) \geq [\frac{\partial\tau}{\partial\theta} + \frac{\partial}{\partial\theta}B^*(\hat{\tau}_V \mid \tau)]^2[I(\theta)]^{-1} \tag{9.3}$$

and

$$E[(\hat{\tau}_V - \tau)^2 \mid \tau] \geq [\frac{\partial\tau}{\partial\theta} + \frac{\partial}{\partial\theta}B^*(\hat{\tau}_V \mid \tau)]^2[I(\theta)]^{-1} + [B^*(\hat{\tau}_V \mid \tau)]^2 \ . \tag{9.4}$$

The reciprocals of the right hand sides of the above two inequalities provide us with the two modified test information functions for the transformed latent variable τ , i.e.,

$$\Upsilon^*(\tau) = I(\theta)\ [\frac{\partial\tau}{\partial\theta} + \frac{\partial}{\partial\theta}B^*(\hat{\tau}_V \mid \tau)]^{-2} \ , \tag{9.5}$$

and

$$\Xi^*(\tau) = I(\theta)\ [\{\frac{\partial\tau}{\partial\theta} + \frac{\partial}{\partial\theta}B^*(\hat{\tau}_V \mid \tau)\}^2 + I(\theta)\ \{B^*(\hat{\tau}_V \mid \tau)\}^2]^{-1} \ . \tag{9.6}$$

In the general case of discrete item responses we can write for the MLE bias function $B^*(\hat{\tau}_V \mid \tau)$ and its derivative with respect to θ

$$\begin{aligned} B^*(\hat{\tau}_V \mid \tau) &= B(\hat{\theta}_V \mid \theta)[\frac{\partial\theta}{\partial\tau}]^{-1} - (1/2)[I(\theta)]^{-1}[\frac{\partial\theta}{\partial\tau}]^{-3}\ \frac{\partial^2\theta}{\partial\tau^2} \\ &= B(\hat{\theta}_V \mid \theta)\frac{\partial\tau}{\partial\theta} + (1/2)[I(\theta)]^{-1}\frac{\partial^2\tau}{\partial\theta^2} \ , \end{aligned} \tag{9.7}$$

and

$$\begin{aligned} \frac{\partial}{\partial\theta}B^*(\hat{\tau}_V \mid \tau) &= B(\hat{\theta}_V \mid \theta)\ \frac{\partial^2\tau}{\partial\theta^2} + [\frac{\partial}{\partial\theta}B(\hat{\theta}_V \mid \theta)]\ \frac{\partial\tau}{\partial\theta} \\ &\quad +(1/2)[I(\theta)]^{-2}[I(\theta)\ \frac{\partial^3\tau}{\partial\theta^3} - I'(\theta)\ \frac{\partial^2\tau}{\partial\theta^2}] \ , \end{aligned} \tag{9.8}$$

respectively (cf. Samejima, 1987). Thus we can use (9.7) and (9.8) in evaluating the modified test information functions, $\Upsilon^*(\tau)$ and $\Xi^*(\tau)$, which are given by (9.5) and (9.6).

X Discussion and Conclusions

A minimum bound of any estimator, biased or unbiased, is considered, and, based on that, Modification Formula No. 1 is proposed for the maximum likelihood estimator, in place of the test information function. A minimum bound of the mean squared error is considered, and, based on that, Modification Formula No. 2 in the same context is proposed. Examples are given, and the usefulnesses of these modified test information functions in computerized adaptive testing are discussed. These topics are also discussed and observed for the monotonically transformed latent variable.

It is expected that these two modification formulae of the test information function can effectively be used in order to supplement deficiencies of the test information function in different situations. Results are yet to come, and the author hopes that other researchers will also use these functions in their own research and observe their effectiveness.

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